DLKoopman: A deep learning software package for Koopman theory

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Created at Galois for DARPA program Symbiotic Design for Cyber-Physical Systems (SDCPS)

pip install dlkoopman

https://github.com/GaloisInc/dlkoopman

Docs: https://galoisinc.github.io/dlkoopman/

dlkoopman

Koopman theory

Data-driven analysis of dynamical systems.

Learn a linear model for a generally nonlinear system only from its snapshots (states).

Encode the states into a different domain using an encoder g. $\boldsymbol{y} = g(\boldsymbol{x})$

Learn the **linear evolution rule** in this domain via the Koopman matrix K. $\boldsymbol{y}_i = \boldsymbol{K}^i \boldsymbol{y}_0$

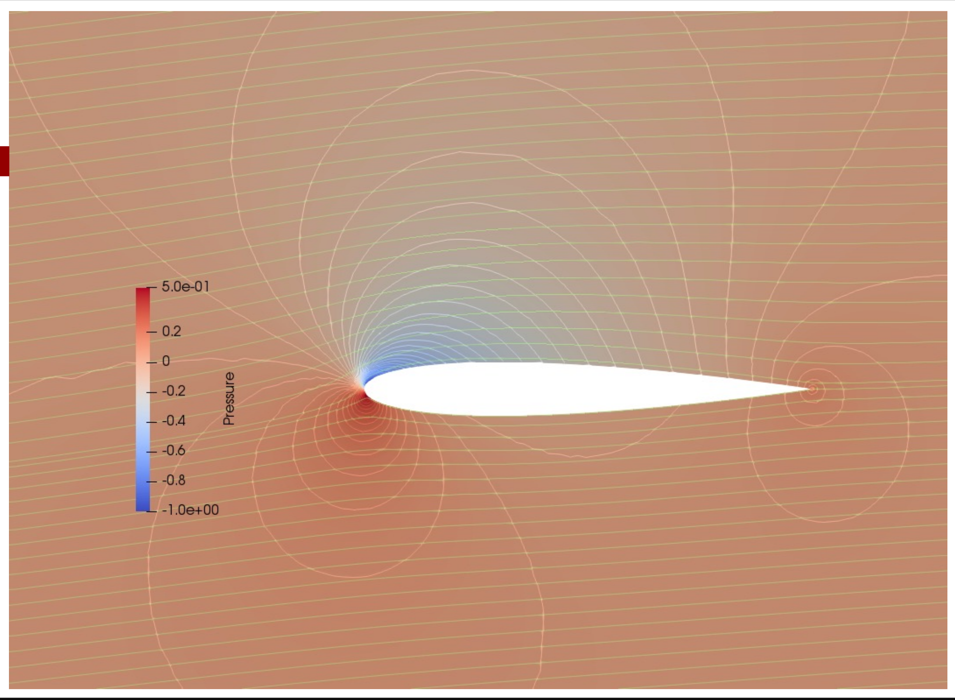
Decode back into the original domain using a decoder g^{-1}. $\boldsymbol{x} = g^{-1}(\boldsymbol{y})$

Example Application

1 m/s flow around the cross-section of a NACA0012 foil in water at 5° angle of attack.

- Pressure contour lines → white
- Streamlines → pale green

Koopman theory can learn the pressure vector from given index values = angles of attack, then predict it for unknown angles of attack, e.g. 50°, 0.5°, or -2°.



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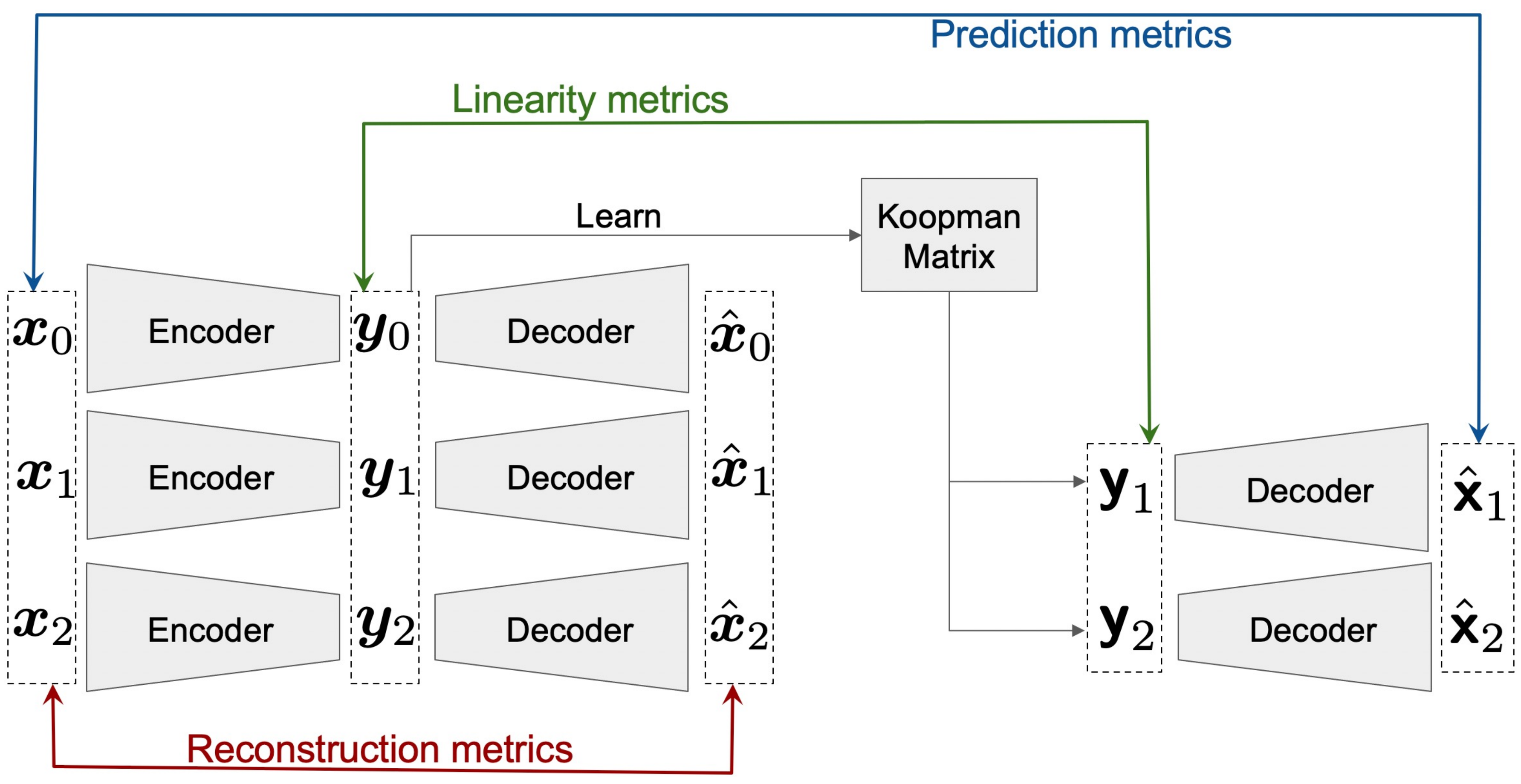
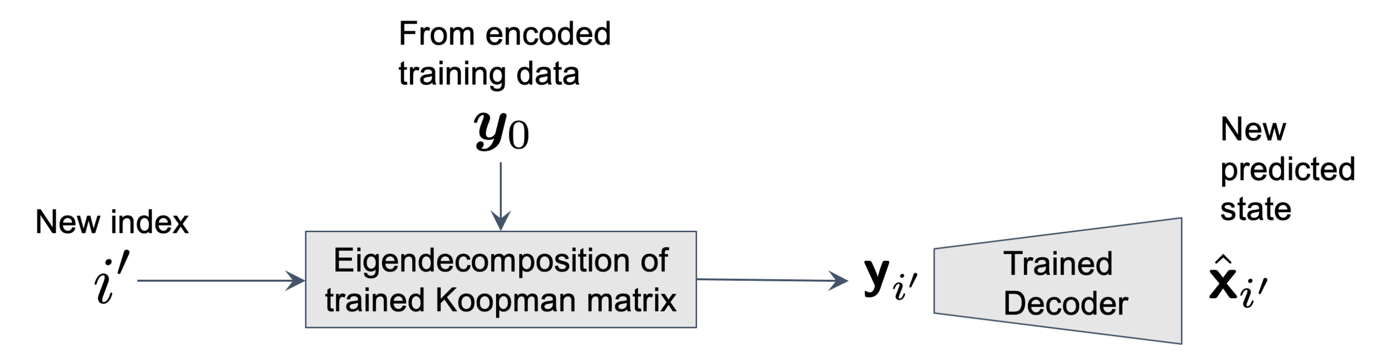


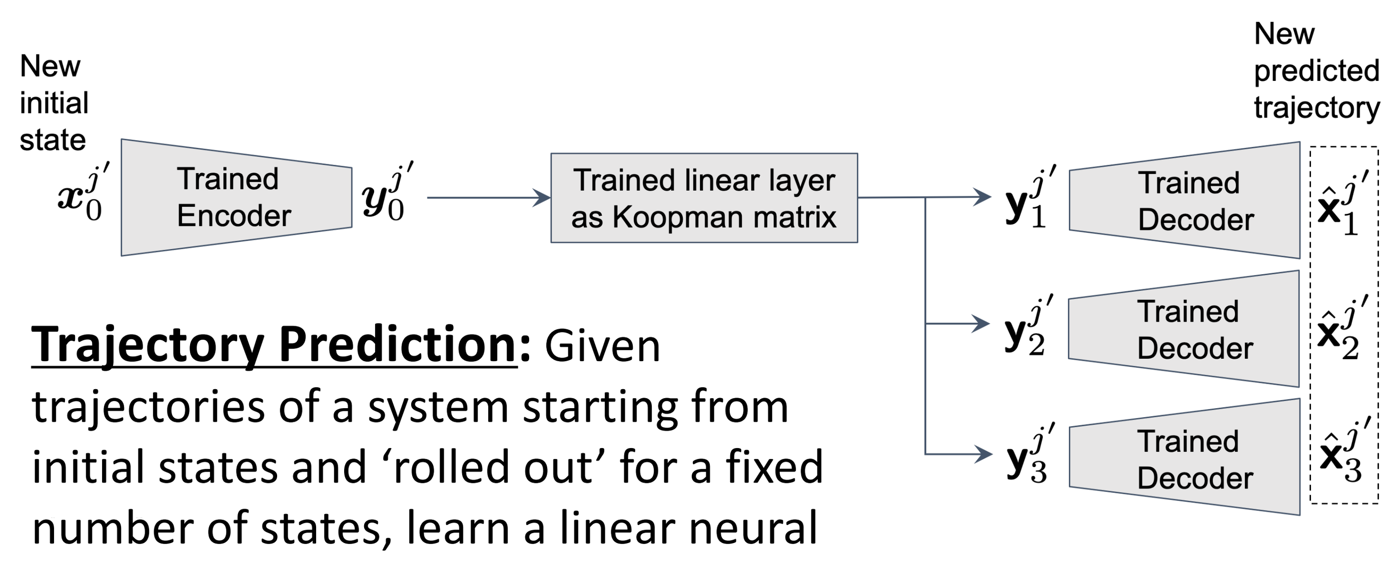
- Reconstruction: How well can the decoder learn the inverse of the encoder?
- Linearity: How good is the linear approximation in the encoded domain?
- Prediction: How good is the overall pipeline at predicting unknown states?

Training loss

$$L = L_{\text{lin}} + \alpha\left(L_{\text{recon}} + L_{\text{pred}}\right) + \beta L_{\text{Autoencoder}} + \gamma L_{\boldsymbol{K}}$$

State Prediction: Given states of a system, learn the Koopman matrix and use its eigendecomposition to predict unknown states in the future, past, or in-between.





Trajectory Prediction: Given trajectories of a system starting from initial states and 'rolled out' for a fixed number of states, learn a linear neural net as the Koopman matrix and use it to predict new trajectories from new initial states.

Example Usage (Python)

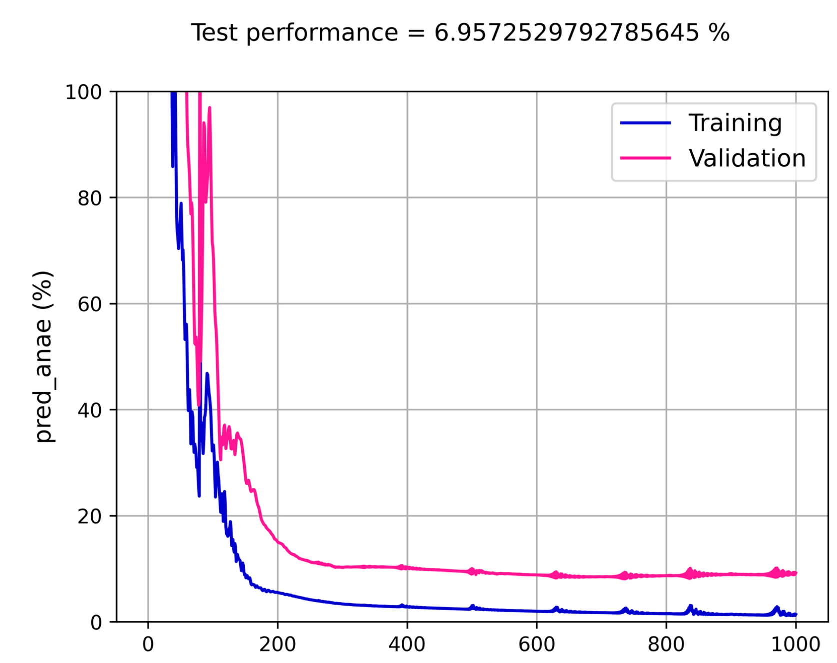
```python
## Define the model
sp = StatePred(
    dh = dh, #StatePredDataHandler instance used to pass data
    rank = 6, #rank of the Koopman matrix K
    encoded_size = 50, #dimensionality of y
    encoder_hidden_layers = [100] #number of neurons
) #above values can be selected via hyperparameter search

## Train the model
sp.train_net(
    numepochs = 1000,
    decoder_loss_weight = 0.1, #alpha in the loss equation
    weight_decay = 1e-5, #beta in the loss equation
    Kreg = 0 #gamma in the loss equation
) #above values can be selected via hyperparameter search

## Perform new predictions
sp.predict_new([3.75,21]) #unknown state indexes
```

Additional Features

Hyperparameter search module to iterate through network sizes and training parameters and select the best configuration(s).

Average Normalized Absolute Error (**ANAE**)

$$\text{ANAE}(\boldsymbol{p}, \boldsymbol{q}) = \text{Avg}_{p_i \neq 0}\left(\frac{|p_i - q_i|}{|p_i|}\right)$$



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